Math 3150 Exam 3D, April 2026, 12 problems, 75 minutes Name: Answers

Read these instructions:

- Leaving the testing room results in a new exam given for the unfinished problems.
- Two detached sheets of notes allowed.
- No electronics.
- Raise your hand for questions or more paper.

Problem 1. Write the following statements in terms of the letters $m =$ "Jo is a math major", $c =$ "Jo is a CS major", $d =$ "Jo is a DS major" and the symbols $\neg$, $\wedge$, $\vee$, $\oplus$.

+2 **Part A.** "Jo is not a math major." $\neg m$

+2 **Part B.** "Jo is neither a CS major nor a DS major, but Jo is a math major." $(\neg c) \wedge (\neg d) \wedge (m)$

+2 **Part C.** "Jo is either a CS major or a math major." $c \vee m$ OR $c \oplus m$.

+2 **Problem 2A.** State the **converse** of "If I don't sleep, then I don't dream" in English.

If I don't dream, then I don't sleep

+2 **Problem 2B.** State the **contrapositive** of "If I don't sleep, then I don't dream" in English.

If I dream, then I sleep

+3 **Problem 3A.** Write the truth table for $(p \vee q) \Rightarrow q$.

p	q	$p \vee q$	$(p \vee q) \Rightarrow q$
T	T	T	T
T	F	T	F
F	T	T	T
F	F	F	T

+2 **Problem 3B.** Find a disjunctive normal form for $(p \vee q) \Rightarrow q$.

$(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$ ... alternate answer: $\neg p \vee q$.

+5 **Problem 4.** Is $(p \Rightarrow q) \wedge q$ logically equivalent to $q \Rightarrow p$? Explain.

p	q	$p \Rightarrow q$	$(p \Rightarrow q) \wedge q$	$q \Rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

last 2 columns are not the same, so NO.

+2 **Problem 5.** Let $T(x)$ ="x has thorns", $P(x)$ ="x has petals", and R be the set of all roses. Write "If some rose h thorns, then every rose has petals" in terms of x, $T(x)$, $P(x)$, R, $\forall$, $\exists$, $\in$, :, [,], $\wedge$, $\vee$, $\neg$, $\Rightarrow$.

$$[\exists x \in R : T(x)] \Rightarrow [\forall x \in R : P(x)]$$

+6 **Problem 6.** Let $B(x)$ = "x has at least one bird", $C(x)$ = "x has at least one cat", $D(x)$ = "x has at least one dog and S be the set of students in this class. Formalize (a)-(c) via the symbols $S, x, B(x), C(x), D(x), \wedge, \vee, \neg, \Rightarrow, \forall, \exists, :$ (colon), $\in, [,]$.

+2 **Part A.** Some student in this class has at least one bird, at least one cat, or at least one dog.

$\exists x \in S : B(x) \vee C(x) \vee D(x)$

+2 **Part B.** Every student in this class has at least one cat and no dog.

$\forall x \in S : C(x) \wedge \neg D(x)$

+2 **Part C.** Some student in this class has at most one type of the animals out of birds, cats, and dogs.

$\exists x \in S : [B(x) \wedge \neg C(x) \wedge \neg D(x)] \vee [\neg B(x) \wedge C(x) \wedge \neg D(x)] \vee [\neg B(x) \wedge \neg C(x) \wedge D(x)] \vee [\neg B(x) \wedge \neg C(x) \wedge \neg D($

+2 **Problem 7.** Which option below is equivalent to the **negation** of "Mo is either tall or slim."?

not tall and not slim

(a) Mo is tall, and Mo is slim.
(b) Mo is not tall, and Mo is not slim.
(c) Mo is tall, or Mo is slim.
(d) Mo is not tall, or Mo is not slim.

Problem 8. Mark each item as true or false. Justify your answers.

+2 **Part A.** $\exists x \in \mathbb{Q} : x^2 < x$ True: let $x = \frac{1}{2}$. Then $(\frac{1}{2})^2 = \frac{1}{4} < \frac{1}{2}$.

+2 **Part B.** $\exists x \in \mathbb{R} : [\forall y \in \mathbb{R} : x + y^2 = 0]$ False: if such x exists making $x + y^2 = 0$, then choosing $(y \pm 1)$ as new y gives $x + (y \pm 1)^2 = x + y^2 \pm 2y + 1 = 0 \pm 2y + 1$. But $\pm 2y + 1$ can't be both zero.

+2 **Part C.** $\forall x \in \mathbb{R} : [\exists y \in \mathbb{R} : x + y^2 = 0]$

False: if $x = +1$, then $+1 + y^2 = 0 \Rightarrow y^2 = -1$ has no real solution bc $y^2 \geq 0$ for all $y \in \mathbb{R}$.

+2 **Part D.** $\forall y \in \mathbb{R} : [\exists x \in \mathbb{R} : xy > 0]$

False: for $y = 0$, $x \cdot y = x \cdot 0 = 0 \not> 0$ no matter the $x \in \mathbb{R}$.

+2 **Part E.** $\exists x \in \mathbb{R} : [(x > 0) \Rightarrow (x^2 = -1)]$

True: choose $x = 0$. Then $[(x > 0) \Rightarrow (x^2 = -1)] = [F \Rightarrow F] = T$.

Problem 9. Is the function $f : \{a, b, c, d\} \to \{u, v, w, x, y\}$ defined by $f(a) = y$, $f(b) = x$, $f(c) = w$, $f(d) = y$ onto? Explain.

+2

No: u is not an output of f.

Problem 10A. Is the function $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = 2(x + 1)$ onto? Justify your answer.

+4

Yes: let $y \in \text{codomain}(g) = \mathbb{R}$. Then $y = g(x)$

$$\Rightarrow y = 2(x+1) \overset{\div 2}{\Rightarrow} \frac{y}{2} = x + 1 \overset{-1}{\Rightarrow} x = \frac{y}{2} - 1 \in \mathbb{R} = \text{domain}(g)$$

So g is onto.

Problem 10B. Is the function $h : \mathbb{Z} \to \mathbb{Z}$ defined by $h(x) = 2(x + 1)$ one-to-one? Justify your answer.

+4

Yes: $h(x_1) = h(x_2) \Rightarrow 2(x_1+1) = 2(x_2+1) \overset{\div 2}{\Rightarrow} x_1 + 1 = x_2 + 1 \overset{-1}{\Rightarrow} x_1 = x_2$

So h is one-to-one.

Problem 11. Suppose that p and q are statements such that $p \Rightarrow q$ is false. $\boxed{\text{only occurs when } p = T, q = F.}$

+2

Part A. Find the truth value of $p \wedge q$. Justify your answer.

$p \wedge q = T \wedge F = F$.

+2

Part B. Find the truth value of $\neg p \vee q$. Justify your answer.

$\neg p \vee q = \neg T \vee F = F \vee F = F$.

Problem 12. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(n) = n^2$.

+3

Part A. Is the statement $\underbrace{\forall x \in \mathbb{R} : [\forall y \in \mathbb{R} : (f(x) = f(y)) \Rightarrow (x = y)]}_{\text{"f is one-to-one"}}$ true or false? Justify your answer.

False: $f(1) = 1^2 = 1 = (-1)^2 = f(-1)$ but $1 \neq -1$ so f is not one-to-one.

+3

Part B. Is the statement $\underbrace{\forall y \in \mathbb{R} : [\exists x \in \mathbb{R} : y = f(x)]}_{\text{"f is onto"}}$ true or false? Justify your answer.

False: $f(n) = n^2 \geq 0$ for all real # n so -1 is not an output of f so f is not onto.

Score	20	20	20